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Some Useful Formulas

1. **Forward Pricing and Value**:
$$F_t = S_t e^{r(T-t)} = S_t(1+R); \qquad V_t = (F_t - F_0)e^{-r(T-t)}, \quad \text{(long/buy position)};$$
where r and R are the continuous and simple interest rates, respectively over time $[t, T]$. In general,
$$F_t = (S_t + C - Y)e^{r(T-t)} = (S_t + C - Y)(1+R),$$
where C is the PV of cost of carry and Y the value of convenience yield. For stocks or stock indices, $F_t = (S_t - D)e^{r(T-t)} = S_t e^{(r-d)(T-t)}$, where D is the PV of dividends and d the dividend yield rate.

2. **Optimal Hedge Ratio**:
$$h = -\rho_{SF}\frac{\sigma_S}{\sigma_F} = -\frac{\sigma_{SF}}{\sigma_F^2} = -\alpha_1 = -\beta,$$
where σ_S and σ_F are the stds, ρ_{SF} and σ_{SF} are the corr. and cov.; and β are slopes of
$$\text{price regression}: \qquad \Delta_{S,t} = \alpha_0 + \alpha_1\Delta_{F,t} + \epsilon_t, \quad t = 1, \ldots, T,$$
$$\text{return regression}: \qquad \tilde{R}_{pt} - r = \alpha + \beta(\tilde{R}_{\text{S\&P500,t}} - r) + \tilde{\epsilon}_t, \quad t = 1, \ldots, T.$$
R^2 assesses the effectiveness of hedging; $N_F = -hN_S$ or $hN_S \times P/F = h \times$ value of port/futures price (N_F should be divided by contract size in practice, and be reduced if there is natural hedge).

3. **Forward Rate and Yields**:
$$(1+R_2)^2 = (1+R_1)(1+f_1); \qquad \text{PV} = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \cdots + \frac{C}{(1+y)^T} + \frac{F}{(1+y)^T}.$$

4. **Duration and Convexity**:
$$\frac{dB}{B} \approx -D_m \times dy + \text{Convexity} \times (dy)^2,$$
i.e., % change of a bond's value is approximately the modified duration times yield change (1st-order accuracy), and $-D_m \times dy$ plus Convexity times yield change squared (2nd-order accuracy), where
$$D_m = \frac{D}{1+y}, \quad D = \left[\frac{1\times C}{1+y} + \frac{2\times C}{(1+y)^2} + \cdots + \frac{n\times C}{(1+y)^n} + \frac{n\times F}{(1+y)^n}\right]/B.$$

5. **Hedging Bonds by Duration** :
$$h = \frac{P(D^* - D_P)}{D_F F}, \qquad (D^* \text{ is the desired duration}).$$

6. **Currency Forwards Pricing** :
$$F_t = S_t e^{(r_d - r_f)(T-t)} = S_t\frac{1+R_d}{1+R_f}; \qquad E(S_T) = F_t \approx S_t\frac{1+I_d}{1+I_f},$$
where r_d and r_f are the domestic and foreign risk-free interest rates (interest rate parity and PPP).

7. **Put–Call Parity**:
$$S + P = C + \text{PV}(X), \qquad \text{PV}(X) = Xe^{-rT} = X/(1+R).$$

8. **Binomial Model**:
$$S_u = (1+u)S, \quad S_d = (1+d)S; \qquad p\times(1+u)S + (1-p)\times(1+d)S = (1+r)S$$
$$p = \frac{r-d}{u-d}; \quad C = \frac{pC_u + (1-p)C_d}{1+r}; \quad \Delta = \frac{C_u - C_d}{S(u-d)}, \quad B = \frac{(1+u)C_d - (1+d)C_u}{(u-d)(1+r)}.$$

9. **Black-Scholes Formula**:
$$C = S\,N(d_1) - X\,e^{-rT}N(d_2), \quad \text{Long Call} = \text{Long Underlying Asset} + \text{Borrowing (RePort)};$$
$$P = X\,e^{-rT}N(-d_2) - S\,N(-d_1); \quad d_1 = [\ln(S/X) + (r + \sigma^2/2)\,T]/\sqrt{\sigma^2 T}, \quad d_2 = d_1 - \sqrt{\sigma^2 T}.$$