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Finance 524B
Derivatives

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Some Useful Formulas

1. **Put–Call Parity**:

$$S + p = c + \mathrm{PV}(X), \quad Se^{-d(T-t)} + p = c + \mathrm{PV}(X), \quad S + p = c + D + \mathrm{PV}(X)$$

$$S - X \le C - P \le Se^{(b-r)T} - Xe^{-rT},\ (b \ge r); \quad Se^{(b-r)T} - X \le C - P \le S - Xe^{-rT},\ (b < r)$$

2. **Black-Scholes Formula**:

$$c = Se^{(b-r)T} N(d_1) - X\, e^{-rT} N(d_2), \qquad p = X\, e^{-rT} N(-d_2) - Se^{(b-r)T} N(-d_1)$$

$$d_1 = [\ln(S/X) + (b + \sigma^2/2)\, T]/\sqrt{\sigma^2 T}, \quad d_2 = d_1 - \sqrt{\sigma^2 T}$$

(Note: cost of carry $b = r - d$ for stock and stock index options; $r_d - r_f$ for currency options; and 0 for futures options. The European call formula applies also to American call options on stocks that pay no or discrete dividends if S is set as the stock price subtracted out the PV of the dividends.)

3. **Comparative Statics**:

$$\Delta = \frac{\partial C}{\partial S} = e^{(b-r)T} N(d_1) > 0,\ \eta = \Delta \frac{S}{C} = e^{(b-r)T} N(d_1) \frac{S}{C} > 1,\ \gamma = \frac{\partial \Delta}{\partial S} = \frac{n(d_1) e^{(b-r)T}}{S\sigma\sqrt{T}} > 0$$

$$\text{Vega} = \lambda = \kappa = \frac{\partial C}{\partial \sigma} = S\sqrt{T} e^{(b-r)T} n(d_1) > 0,\ \rho = \frac{\partial C}{\partial r} = T\, X e^{-rT} N(d_2) > 0,\ \theta = \frac{\partial C}{\partial T} > 0\ (\text{no}-\text{div})$$

4. **Binomial Model**:

$$p \times (1+u)S + (1-p) \times (1+d)S = (1+r)S, \quad \text{or} \quad p = \frac{r-d}{u-d}; \quad c = \frac{pc_u + (1-p)c_d}{1+r}$$

$$C_i = \max\left[S_i - X, \frac{pC_{iu} + (1-p)C_{id}}{1+r}\right], \quad P_i = \max\left[X - S_i, \frac{pP_{iu} + (1-p)P_{id}}{1+r}\right]$$

   a) To transfer a continuous-time model with σ, r and T into an n-period binomial model, set:

$$u = e^{\sigma\sqrt{\Delta T}} - 1, \quad d = e^{-\sigma\sqrt{\Delta T}} - 1, \quad r^* = e^{r\Delta T} - 1, \quad \Delta T = \frac{T}{n}, \quad p = \frac{r^* - d}{u - d}$$

   b) Corporate (ex-coupon) bond values:

$$B_{\text{node}} = \min(V_{\text{node}} - c, F),\ (\text{at } T); \quad B_{\text{node}} = \frac{p \times B_{\text{node-u}} + (1-p) \times B_{\text{node-d}} + c}{1+r}$$

5. **Value at Risk** $N(-2.33) = 1\%$, $N(-1.645) = 5\%$, 99% and 95% N day VaRs are, respectively,

$$\text{VaR} = 2.33 \times \text{vol of value change}, \ \text{VaR} = 1.645 \times \text{vol of value change}$$

$$\sigma_{AB} = \sqrt{\sigma_A^2 + \sigma_B^2 + 2\rho\sigma_A\sigma_B}, \quad \sigma_P^2 = \sum_{i=1}^{N} \sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j=1}^{i-1} \sigma_i\sigma_j$$

6. **Replicating Portfolios**: (the key insight in option pricing)

$$\text{Call} = \text{Long Underlying Asset} + \text{Borrowing}$$

$$C = \Delta\, S + B, \quad \Delta = \frac{C_u - C_d}{S(u-d)}, \quad B = \frac{(1+u)C_d - (1+d)C_u}{(u-d)(1+r)}$$

(Note: the replicating portfolios are given by the Black-Scholes formula in continuous time.)

7. **Ito's Lemma**:

$$dx_t = a(x,t)\, dt + b(x,t)\, dB_t$$

$$\Rightarrow \quad dG(x,t) = \left[G_t + aG_x + \frac{1}{2} b^2 G_{xx}\right] dt + bG_x\, dB_t$$